

DTP-97/25 February 1, 2008

Skyrmions and domain walls in (2+1) dimensionsA. Kudryavtsev¹

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We study classical solutions of the vector $O(3)$ sigma model in $(2 + 1)$ dimensions, spontaneously broken to $O(2) \times Z_2$. The model possesses Skymion-type solutions as well as stable domain walls which connect different vacua. We show that different types of waves can propagate on the wall, including waves carrying a topological charge. The domain wall can also absorb Skymions and, under appropriate initial conditions, it is possible to emit a Skymion from the wall.

1. Introduction.

In our previous works [1-3] various solutions of the so-called baby-Skymion model were investigated. In this paper we begin a systematic study of solutions of the nonlinear vector $O(3)$ sigma model spontaneously broken to $O(2) \times Z_2$.

The (2+1)-dimensional version of this model is described by the Lagrangian density

$$L = F_\pi \left(\frac{1}{2} \partial_\alpha \vec{\phi} \partial^\alpha \vec{\phi} - \frac{k^2}{4} (\partial_\alpha \vec{\phi} \times \partial_\beta \vec{\phi}) (\partial^\alpha \vec{\phi} \times \partial^\beta \vec{\phi}) - \frac{\mu^2}{2} (1 - \phi_3^2) \right). \quad (1.1)$$

Here $\vec{\phi} \equiv (\phi_1, \phi_2, \phi_3)$ denotes a triplet of scalar real fields which satisfy the constraint $\vec{\phi}^2 = 1$; $(\partial_\alpha \partial^\alpha = \partial_t \partial_t - \partial_x \partial_x - \partial_y \partial_y)$.

The first term in (1.1) is the familiar Lagrangian density of the pure S^2 σ -model. The second term, fourth order in derivatives, is the (2+1) dimensional analogue of the Skyrme-term of the three-dimensional Skyrme-model. The last term is often referred to as a potential term. The last two terms in the Lagrangian (1.1) are added to guarantee the stability of a Skymion [4]. In contradistinction to the first two terms the potential term in (1.1) differs from that of the baby-Skymion model [1-3] and is quadratic in terms of the field ϕ_3 . Our potential looks like a mass term for the ϕ_1 and ϕ_2 fields but, unlike the baby-Skymion potential, it does not provide any additional interaction between them. Its normalisation was chosen so that in the limit of ϕ_3 being close to 1 both the baby-Skymion potential, $\mu^2(1 - \phi_3)$, and our potential are identical and approximated by $\frac{\mu^2}{2}(\phi_1^2 + \phi_2^2)$.

As in [1-3] we fix our units of energy and length by setting $F_\pi = k = 1$ and we choose $\mu^2 = 0.1$ to compare our results with what was observed in [1].

Notice, that the gauge version of the Lagrangian (1.1) was recently studied in [5] (without the Skyrme term). Some aspects of the soliton stability for the model (1.1) were also discussed in [6].

Let us add that our model (corresponding to the Lagrangian density (1.1)) can be derived in the continuous limit of easy-axis Heisenberg antiferromagnetics [7] and ferroelectrics with an easy-axis anisotropy [8] (without the Skyrme term).

One clearly sees from (1.1) that the model has two distinct vacua: $\phi_3 = \phi_\pm = \pm 1$. If we require that the field at spatial infinity describes one of the two vacua, we can compactify the two dimensional plane into a sphere making the theory topological. All classical configurations will be characterised by the value of the winding number

$$N = \frac{1}{8\pi} \int \varepsilon_{ij} \vec{\phi} (\partial^i \vec{\phi} \times \partial^j \vec{\phi}) d^2 x \quad (1.2)$$

which is an integer as it counts the number of times the target sphere S^2_{iso} is covered by the physical space S^2_{space} .

As we will demonstrate our model possesses two types of solutions, i.e. Skyrmions, and domain walls represented by a (1+1) dimensional soliton. Domain walls connect different vacua ϕ_\pm of the theory making the interaction between a Skyrmion and the domain walls nontrivial.

The interaction of particles with domain walls has become the subject of interest beginning with the paper of Voloshin [9]. A detailed analysis of the scattering of gauge Abelian particles on domain walls may be found in a recent paper by Farrar et al. [10]. The physics of fermion zero modes on strings and domain walls was discussed widely in literature, especially in connection with the problem of anomalies, see e.g. [11] and refs. therein. The problem of the chiral fermion determinant in the presence of a domain wall was discussed recently in [12]. It is also worth mentioning that the expanding bubbles of a new phase during the electroweak phase transition may be an additional source for the CP-violation effects and for the electroweak baryogenesis, see e.g. [13,14].

In this paper we consider a slightly different aspect of the interaction of particles with domain walls. Namely, we study the interaction of two objects of our model, i.e. a Skyrmion and a domain wall. We demonstrate that this interaction is nontrivial and that it leads to the absorption of the Skyrmion by the wall. The conserved topological charge of the Skyrmion is transferred to the waves on the wall which then carry a topological charge. Under certain initial conditions the collision of the waves of the wall stimulates the emission of an isolated Skyrmion off the wall.

2. Travelling waves on the domain wall

First of all, we will study soliton like solutions of the theory (1.1) . To do this it is convenient to change variables and to use two real fields f and ψ instead of $\vec{\phi}$,

$$\vec{\phi} = (\sin f \cos \psi, \sin f \sin \psi, \cos f) \quad (2.1),$$

where $f \in [0, \pi]$ and $\psi \in [0, 2\pi]$. In terms of f and ψ the Lagrangian (1.1) has the following form

$$L = \frac{1}{2}(f_t^2 - f_x^2 - f_y^2) + \frac{\sin^2 f}{2} [(f_t \psi_x - f_x \psi_t)^2 + (f_t \psi_y - f_y \psi_t)^2 - (f_x \psi_y - f_y \psi_x)^2] - \frac{\mu^2}{2} \sin^2 f. \quad (2.2)$$

First we look for one dimensional static solutions of the equations of motion for (2.2) in the form $f = f(x)$, $\psi = \text{const.}$ Then the equation for f reduces to

$$f_{xx} - \frac{\mu^2}{2} \sin 2f = 0, \quad (2.3)$$

which is a static version of the sine-Gordon equation with the soliton solution

$$f_s(x) = 2 \arctan \exp(\pm \mu(x - x_0)). \quad (2.4)$$

This solution links the two vacua states ϕ_{\pm} of the theory and looks like a wall in the (x, y) -plane, which separates domains of different vacua. In contradistinction to the baby-Skyrmion model studied in previous papers [1-3], in the present model the soliton (2.4) is stable, see also [6]. Its energy is $E = 2\mu = 0.2$ per unit length of the wall.

Notice that our soliton solution (2.4) does not satisfy our boundary conditions for the field at spatial infinity which we introduced to split all field configurations into integer valued classes labelled by the topological charge N . So it may appear that we cannot use the topological concepts when looking at fields such as (2.4). However, this is not the case. We can always consider the solution (2.4) as having come, cut out and expanded, from a field configuration in the shape of a long straight wall whose ends are connected by a semicircular ring of an appropriate large radius.

Using the expression (1.2) and restricting the integration to a region of space we can define the topological charge for the field in a given region. Then, calculating the topological charge for the straight part of the wall we find that it is zero. We will show later that the wall (2.4) can be modified so as to carry a non-zero topological charge.

The domain wall acts like a wave carrier. To study waves carried by the wall we start by looking at infinitesimal perturbation of the solution (2.4) of the form

$$f(x, y, t) = f_s(x) + g(x, y, t) \quad (2.5)$$

where $|g(x, y, t)| \ll 1$ and where f is given by (2.4). The linearised equation of motion for $g(x, y, t)$ is

$$g_{tt} - g_{xx} - g_{yy} + \mu^2 \left(1 - \frac{2}{\cosh^2(\mu x)}\right) g = 0. \quad (2.6)$$

We can look for solutions of the form $g(x, y, t) = \cos(\omega t - ky)\rho(x)$, which correspond to waves propagating along the wall. When $\omega^2 = k^2$ it is more convenient to write the solutions in the form

$$g_0(x, y, t) = \mu [F_1(y - t) + F_2(y + t)] \cosh^{-1}(\mu x), \quad (2.7)$$

where $F_1(z)$ and $F_2(z)$ are arbitrary dimensionless functions of their arguments ($|g_0| \ll 1$). We call such fields zero modes because the x dependence of such solutions corresponds to the zero mode of the sine-Gordon kink. Putting together (2.5) and (2.7) we can write

$$f(x, y, t) = f_s(x) + [F_1(y - t) + F_2(y + t)] \frac{\partial f_s}{\partial x} \quad (2.8)$$

showing that the additional waves correspond to a small displacement of the wall along the transversal (x) direction and which propagate at the speed of light along the wall. These solutions, called travelling waves, were found in [15]. A more general form of the travelling waves in the presence of multivortex configurations was discussed in [16].

Nonzero modes for the equation (2.6) can also be found. They exist for $\omega^2 \geq k^2 + \mu^2$ and correspond to the excitations of the continuum around the soliton (2.4). In terms of waves in the y -direction these excitations look like massive particles. Notice that our model possesses an energy gap between light-like zero modes and massive excitations.

Zero mode excitations of the wall can also be found outside the small amplitude limit $|g| \ll 1$. To see this, we rewrite the equation of motion in terms of the function $f(x, y, t)$ ($\psi = \text{const}$):

$$f_{tt} - f_{xx} - f_{yy} + \frac{\mu^2}{2} \sin 2f = 0. \quad (2.9)$$

The zero-mode solutions of this equation are of the form:

$$f_{\pm}^{(0)}(x, y, t) = f_s(x - x_0^{\pm}(y \pm t)), \quad (2.10)$$

where $x_0^{\pm}(z)$ are arbitrary functions of their arguments. The wave (2.10) represents a transversal displacement of the wall which travels with the velocity of light along the y direction. However, unlike in the linearised case (solutions (2.7) of equation (2.6)), the superposition of solutions $f_+^{(0)}$ and $f_-^{(0)}$ is not a solution of (2.9).

The interaction between two deformation waves is thus non-linear. We have solved numerically the full equation of motion derived from (1.1) to analyse the scattering between 2 kink-like deformations travelling on the wall in opposite directions. The result is shown on Figure 1. One can see from the figures that the collision process is followed by the assymetric emission of waves in the (x, y) plane

t = 0

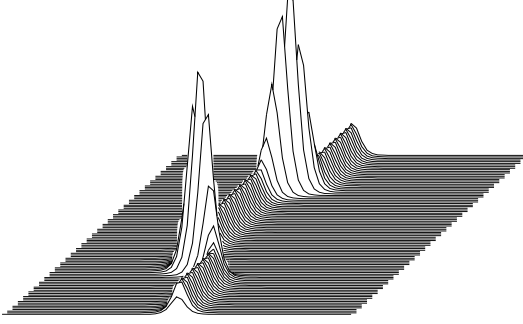


Figure 1.a : Deformation wave : energy density at $t = 0$

t = 30

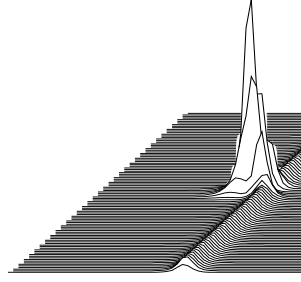


Figure 1.b : Deformation wave : energy density at $t = 30$

t = 52.5

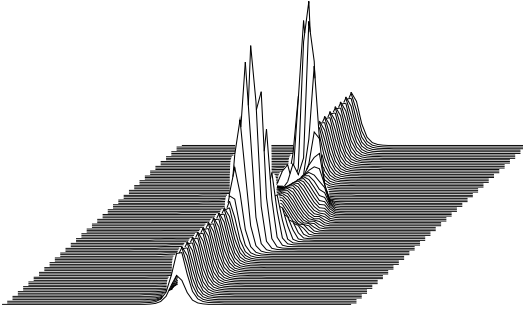


Figure 1.c : Deformation wave : energy density at $t = 52.5$

t = 70.5

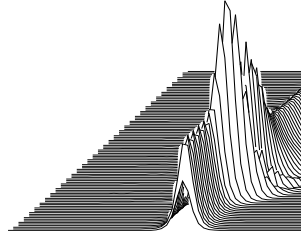


Figure 1.d : Deformation wave : energy density at $t = 70.5$

3. Topological waves on the domain wall

In the previous section, we have derived solutions for which ψ is constant. We will now relax this condition and look for waves involving a change in ψ propagating on the domain wall (2.4) . First of all, looking for solutions of the form $f = f(x)$, $\psi = \psi(y, t)$,

we get the following equation of motion:

$$\begin{aligned} \psi_{tt} - \psi_{yy} &= 0 \\ f_{xx} + \frac{\sin 2f}{2} [(1 - f_x^2)(\psi_t^2 - \psi_y^2) - \mu^2] - \sin^2 f [f_{xx}(\psi_t^2 - \psi_y^2)] &= 0. \end{aligned} \quad (3.1)$$

Solutions of these equations are given by $\psi(y, t) = \psi(y - t)$ or $\psi(y, t) = \psi(y + t)$ and where f is the domain wall solution (2.4). This corresponds to a wave in “ ψ ” propagating on the domain wall in one direction at the speed of light.

Let us return to the boundary conditions for the field $\vec{\phi}$ and the topological charge.

First of all note that the expression for the topological charge written in terms of fields f and ψ is given by

$$N = \frac{1}{4\pi} \int d(\cos f) \wedge d\psi, \quad (3.2)$$

As we have argued before the topological charge carried by our domain wall solution (2.4) is zero. This corresponds to $\psi = 0$. However, thinking of the wall as having come from a closed structure (a straight section and a ring “at infinity”) the total charge of this structure can be an integer m and the charge carried by the wall-like configurations is given by

$$\Delta\psi \Big|_{y=-\infty}^{y=+\infty} = 2\pi\beta, \quad (3.3)$$

where $\beta \in \mathbb{Z}$ if we impose a further condition that the fields at $y \rightarrow \pm\infty$ are the same. This last condition is a very natural generalisation of $\Delta\psi = 0$ satisfied by our domain wall (2.4) and it allows topological waves to propagate along the wall.

Consider, for example, $\psi = \pi[\tanh(y - t) + 1]$, taken together with f given by (2.4) (Fig. 2). This is a solution of (3.1) and it describes a wave propagating on the domain wall. Its topological charge (given by (3.2) or (3.3)) is 1. It is important to note that the topological charge on the wall does not have to be localised; it can be spread out or split into small lumps of fractions of a unit charge.

In Figure 2 we present a snapshot of the solution $\psi = \pi(\tanh(y - t) + 1)$ and where f is given by (2.4). The arrows in fig 2.a correspond to a projection of the S^2 field onto the plane at the equator of the sphere. The orientation of the arrows is thus given by ψ and the length of the vectors is given by $|\sin(f)|$. To distinguish between vectors in the upper or lower hemisphere, we add a “+” or a “ \times ” at the origin of the arrows.

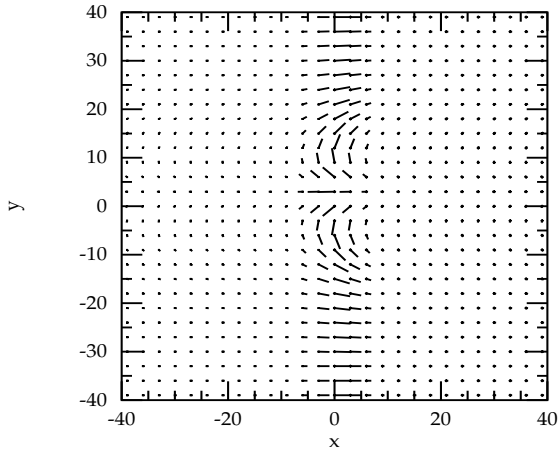


Figure 2.a : Topological wave with total charge $Q = 1$

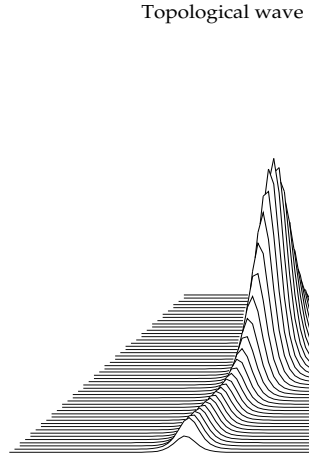


Figure 2.b : Energy of topological wave

The collision between two waves propagating in opposite directions is non-linear and we have solved the equation of motion numerically to perform the scattering of two such waves. The collision is inelastic resulting in the emission of a circular wave as is shown in Figure 3. In contradistinction to the collision of two travelling waves this emission is symmetric.

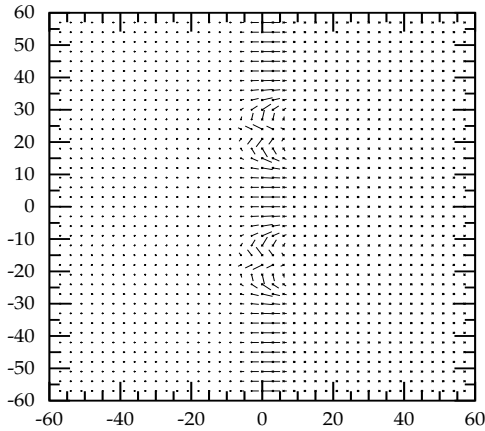


Figure 3.a : Field of topological wave scattering at $t = 0$

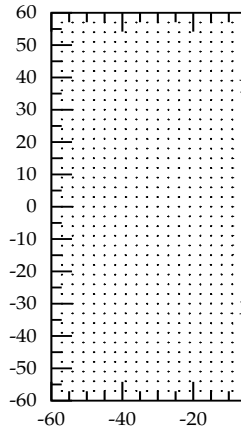


Figure 3.b : Field of topological wave scattering at $t = 60$

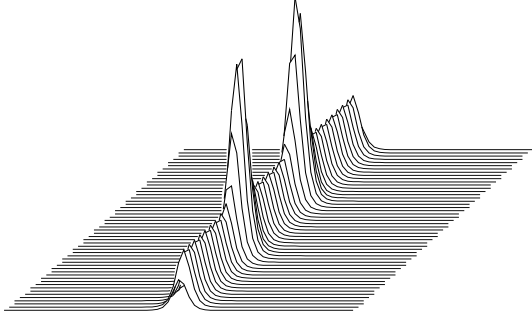


Figure 3.c : Energy density of topological wave scattering at $t = 0$

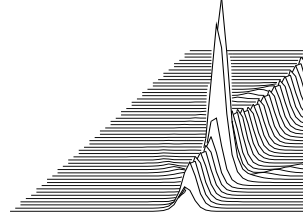


Figure 3.d : Energy density of topological wave scattering

Next, we look again for small amplitude waves and try to find solutions of the form $f(x, y, t) = f_s(x) + g(x, y, t)$, $|g| \ll 1$, $\psi = \psi(y, t)$ linearising the equation of motion in terms of $g(x, y, t)$. We get

$$\begin{aligned} \psi_{tt} - \psi_{yy} &= 0, \\ g_{tt} - g_{xx} - g_{yy} + \mu^2 \left(1 - \frac{2}{\cosh^2 \mu x}\right) g &= \frac{\sinh \mu x}{\cosh^2 \mu x} \left(1 - \frac{2\mu^2}{\cosh^2 \mu x}\right) (\psi_t^2 - \psi_y^2). \end{aligned} \quad (3.4)$$

Notice that the right hand side of this equation is orthogonal to any zero-mode solution for $g(x, y, t)$ (2.6). So topological waves, when they collide, excite only the continuum modes of the soliton. This shows that the topological waves and transversal waves (2.7) can be superimposed. This will play an important role in what follows.

4. Static Skyrmion solution

An important class of static solutions of the equation of motion consists of fields which are invariant under the group of simultaneous spatial rotations by an angle $\varphi \in [0, 2\pi]$ and iso-rotations by $-n\varphi$, around the ϕ_3 axis, where n is a non-zero integer. Such fields are of the form

$$\vec{\phi}(\vec{x}) = \begin{pmatrix} \sin f(r) \cos(n\theta) \\ \sin f(r) \sin(n\theta) \\ \cos f(r) \end{pmatrix}, \quad (4.1)$$

where (r, θ) are polar coordinates in the (x, y) -plane. Such fields are analogues of the hedgehog field of the Skyrme model and were studied in [1-3] for the baby-Skyrmion model.

The function $f(r)$, the analogue of the profile function of the Skyrme model, has to satisfy

$$f(0) = m\pi, \quad m \in Z \quad (4.2)$$

for the field (4.1) to be regular at the origin. As we are looking for field configurations with a finite energy we set

$$\lim_{r \rightarrow \infty} f(r) = l\pi, \quad l \in Z. \quad (4.3)$$

Clearly, Skyrmion solutions with $l = 2k$, $k \in Z$ at $r \rightarrow \infty$ approach the $\phi_3 = \phi_+$ vacuum state and those with $l = 2k + 1$, $k \in Z$ go to the ϕ_- vacuum state. The boundary conditions (4.2) and (4.3) are responsible for making this class of configurations topologically nontrivial. Moreover, the degree of the fields (4.1) (their winding number) is:

$$\deg[\vec{\phi}] = [\cos f(\infty) - \cos f(0)] \frac{n}{2} = [(-1)^l - (-1)^m] \frac{n}{2}. \quad (4.4)$$

The Z_2 symmetry of the model transforms the two types of Skyrmions into one another. We can thus, without any loss of generality, set l to 0. For the case $n = 1$, $m = 1$, $l = 0$, we have $\deg[\vec{\phi}] = 1$. The Euler-Lagrange equation for $f(r)$ is given by

$$\left(r + \frac{n^2 \sin^2 f}{r}\right) f'' + \left(1 - \frac{n^2 \sin^2 f}{r^2} + \frac{n^2 f' \sin f \cos f}{r}\right) f' - \frac{n^2 \sin f \cos f}{r} - r \frac{\mu^2}{2} \sin 2f = 0. \quad (4.5)$$

The profile function $f(r)$ for the case $n = 1$, $m = 1$ and $l = 0$ is shown in Figure 4 (Skyrmion). The case $m = 1, l = 0$ and $n = -1$ corresponds to $\deg[\phi] = -1$, has the same profile function f but it describes an anti-skyrmion. We have found numerically that the total energy of the Skyrmion solution is $E_{Sk} = 1.446$ in our units. This value is smaller than the energy of the baby-Skyrmion ($E = 1.56$) found in [1].

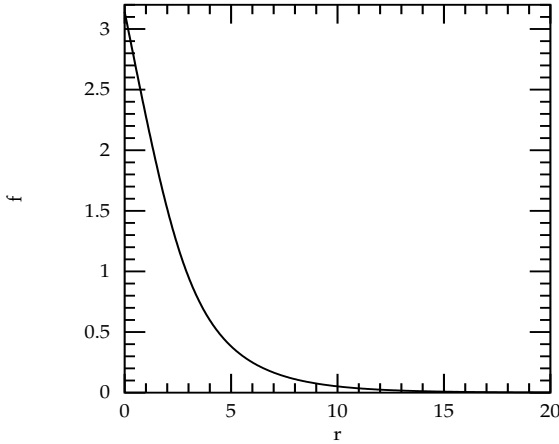


Figure 4.a : Profile function f for the Skyrmion

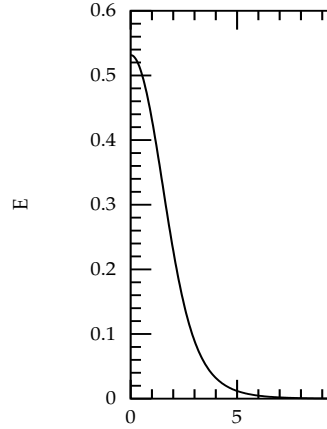


Figure 4.b : Energy of the Skyrmion

5. Skyrmion absorption by domain walls.

So far we have seen that the model described by (1.1) has soliton-like solutions corresponding to domain walls, as well as topological solitons (Skyrmions). We will now study the interaction between these two types of solutions. The force between a domain wall and a Skyrmion depends both on the distance between them and their relative orientation. To superimpose two solutions, we use a stereographic projection of the sphere onto the complex plane and use the field w defined as

$$w = \frac{\phi_1 + i\phi_2}{1 + \phi_3}. \quad (5.1)$$

In this formulation, the vacuum configuration ϕ_+ is given by $w = 0$ and we can in first approximation, superimpose a Skyrmion and the domain wall (on the ϕ_+ side) by adding the 2 fields configurations. As we must also choose a relative orientation between the two structures we can write:

$$w = w_{wall} + e^{i\alpha\pi} w_{Sk}, \quad (5.2)$$

where w_{wall} and w_{Sk} are centered around two points separated by a distance R . In Figure 5 we present a plot of the total energy of the Skyrmion-wall configuration as a function of the separation distance R for different relative orientations α . The energy increase at short distances is due to the fact that at these distances the superposition of the two solutions is not a good approximation to the true configuration. When $\alpha = 0$ the Skyrmion and the wall are in the repulsive channel and they repel. When $\alpha = 1$ the wall attracts the Skyrmion and, we have checked numerically that for other values of α , the Skyrmion and the wall rotate themselves into the attractive channel and move towards each other.

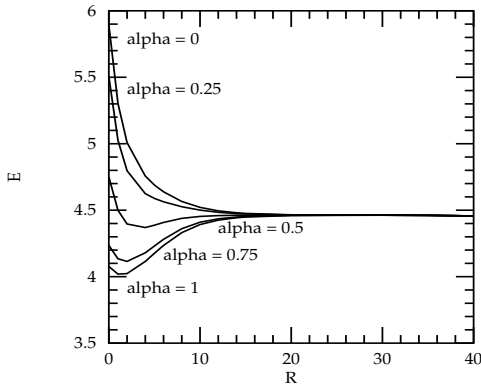


Figure 5 : Skyrmion-wall potential for $\alpha = 0, 0.25, 0.5, 0.75$ and 1.

After the Skyrmion has collided with the wall it splits into two parts which propagate along the wall at the speed of light (Figure 6). Each part carries one half of the unit of topological charge. These two waves are very similar to the superposition of topological and deformation waves given by the solutions of (3.4). The collision is inelastic in the sense that a circular wave is produced in the (x, y) plane implying that the reverse process, of creating a Skyrmion from the wall, is not easy.

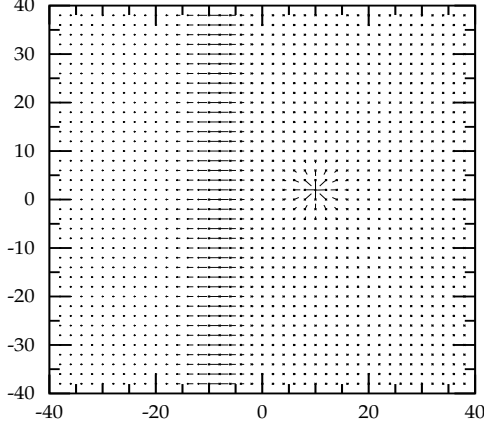


Figure 6.a : Skyrminion-wall field at $t = 0$

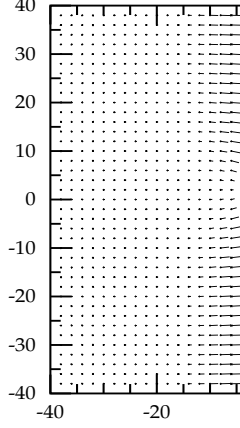


Figure 6.b : Skyrminion-wall field at $t = 123$

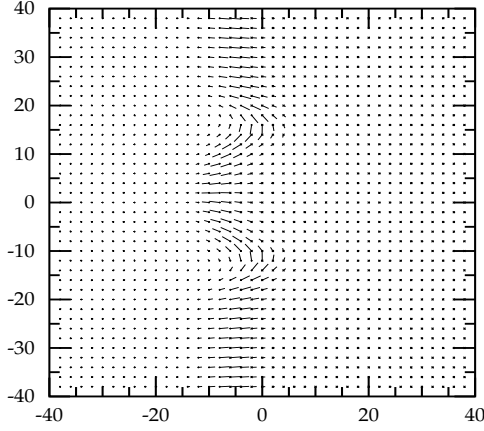


Figure 6.c : Skyrminion-wall field at $t = 135$

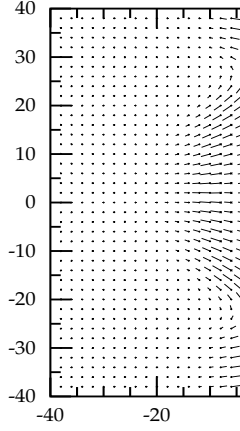


Figure 6.d : Skyrminion-wall field at $t = 145$

6. Creation of a Skyrmion by a domain wall

A domain wall is the lowest energy configuration of the $\vec{\phi}$ -field corresponding to the appropriate boundary conditions. This is why only some excitations of the wall produce particles or waves in the (x, y) -plane. We have already mentioned that the interaction of topological wave packets is a possible source for such a process.

As our first attempt to create a Skyrmion from the wall we put two Skyrmions at a finite distance from each other and made them be absorbed by the same wall at the same time. Each Skyrmion produced two topological waves on the wall; two of these waves collided with each other, creating what looked like a Skyrmion, but there was not enough energy to emit it from the wall.

Looking at the topological wave produced by a Skyrmion when absorbed by a wall, it is clear that we have to combine a topological lump with a deformation wave as described in the third section. Moreover, topological waves are necessary to give a topological charge to the Skyrmion. On the other hand, deformation waves do not carry any topological charge, but these travelling zero-mode play an exceptional role among all excitations of the wall, namely, they preserve the information about the initial deformation of the wall. This is why they can be a source of the energy transfer from the y -direction to the x -direction. This is clearly seen in Figure 1, which shows that such a collision process is strongly anisotropic. The emission of waves in this case is mainly directed to the right from the y -axis. We see in this figure that at a certain time, a jet-like configuration, with an energy peak being emitted along the positive x -axis, is formed. The process is very asymmetric; the overall momentum is conserved by a (small) recoil of the domain wall. Of course, for an infinitely long wall this recoil is negligible.

To create a Skyrmion, we have taken the following initial condition corresponding to the superposition of a deformation and a topological wave

$$w = \exp\left(-\mu\left(x - \frac{B}{2}[\tanh(\mu(y+t-A)) - \tanh(\mu(y-t+A))]\right)\right) \exp\left(\frac{i}{2}\pi(\tanh(\mu(y+t-A-D)) + \tanh(\mu(y-t+A+D)) + 2)\right) \quad (6.1)$$

where A sets the initial distance between the waves packets, B corresponds to the amplitude of deformation (kink) and D introduces a delay between the deformation wave and the topological wave on each packet. (The initial condition is actually given by w and $\frac{dw}{dt}$ both evaluated at $t = 0$). When $D > 0$ the topological wave is retarded with respect to the deformation wave. This is the type of configuration observed in Fig 6d after the wall has absorbed a Skyrmion. For the numerical simulations, we chose $A = 30, B = 20, D = 10$. As is shown in Figure 7, this initial condition results in the emission of a Skyrmion at the right hand side of the wall. After being emitted the Skyrmion moves away from the wall at about half the speed of light and as we can

see from the figure the distance between the wall and the emitted Skyrmion is larger than 20. This, together with the potential shown on Figure 5, clearly indicates that the Skyrmion is detached from the wall.

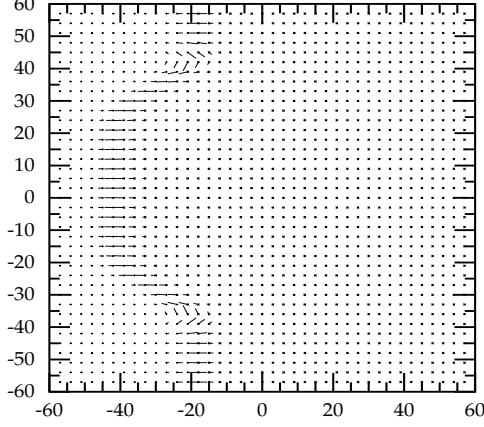


Figure 7.a : Skyrmion emission.
Field at $t = 0$

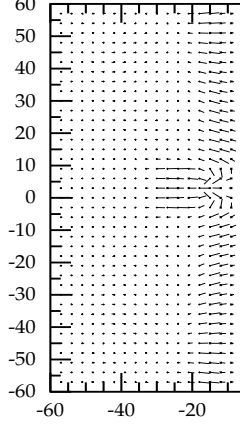


Figure 7.b : Skyrmion emission.
Field at $t = 36$

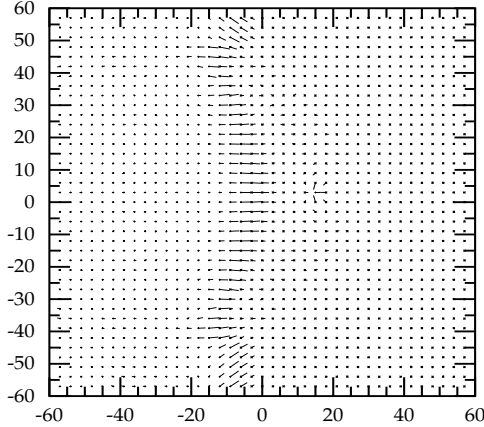


Figure 7.c : Skyrmion emission.
Field at $t = 79.5$

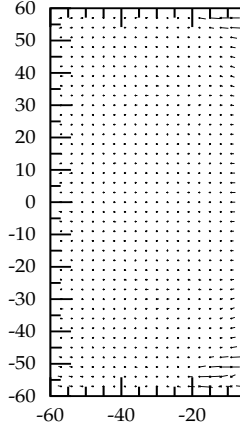


Figure 7.d : Skyrmion emission.
Field at $t = 102$

We have also used other values of D . When D is negative, the Skyrmion is emitted on the other side of the wall, but both the wall and the Skyrmion move to the right after the emission. This suggests that the wall might reabsorb the Skyrmion after a

while (something we are not able to show numerically). When $D = 0$ the wall does not emit any Skyrmion.

The plots in Figure 7 clearly confirm the emission of an isolated Skyrmion by the wall. Moreover, the process is inelastic, i.e. the production of a Skyrmion is accompanied by the emission of mesonic waves.

It is also worth mentioning that in the collision process described above the Skyrmion is created not in its ground state but rather in an excited state. Consequently, the height of the energy density peak of the Skyrmion oscillates around its position of equilibrium (after emission).

7. Conclusions

We have shown that the model described by the Lagrangian density (1.1) has various types of classical solutions including topological solutions (Skyrmions) and domain walls. The domain wall acts as a carrier of waves propagating at the speed of light. Some of these waves have a non-zero topological charge.

Skyrmions and domain walls attract each other leading to the absorption of Skyrmions by the walls and the creation of topological waves. By choosing appropriate initial conditions it is possible to create a Skyrmion from a domain wall. This production takes place for a relatively large range of parameters of these initial conditions.

We hope to be able to apply the observed phenomena to problems of modern high energy physics. The most interesting applications would involve problems of baryogenesis, (see, e.g. paper [13,14] in this connection).

ACKNOWLEDGEMENTS

One of the authors (AK) thanks the Department for Mathematical Sciences of University of Durham for the hospitality during his visit. This visit was supported partly by a UK Royal Society grant, by INTAS grant 93-633 -EXT and partly by grant RFFR-95-02-04681.

The authors want to thank J. Dziarmaga for bringing to their attention the papers [15,16] and him and R.A. Gregory and R.S. Ward for useful comments..

References

1. B.M.A.G. Piette, B.J. Schroers and W.J. Zakrzewski, Z. Phys. C **65** (1995) 165, Nucl. Phys. B **439** (1995) 205
2. B.M.A.G. Piette, H.J.W. Muller-Kirsten, D.H. Tchrakian and W.J. Zakrzewski, Phys.Lett. B **320** (1994) 294
3. A. Kudryavtsev, B. Piette, W.J. Zakrzewski, Durham preprint DTP96/17, Z. Phys. C (1997) to be published.
4. R.A. Leese, M. Peyrard and W.J. Zakrzewski, Nonlinearity **3** (1990) 773
5. I.L. Bogolyubski, A.A. Bogolyubskaya, Phys Lett. **395** (1977) 269

6. Yu-dong Yin, Tao Huang, Jiang-ru Wen. Phys Lett. **373** (1996) 309
7. A.M. Kosevich, B.A. Ivanov and A.S. Kovalev. Phys Reports, **194** (1990) 1
8. J. Pouget and G.A. Maugin. Phys Rev. **B30** (1984) 5306
9. M.B. Voloshin. Sov. J. Nucl. Phys. **21** (1975) 687
10. G.R. Farrar, J.W. McIntosh, Jr. Phys Rev. **D51** (1995) 5889
11. C.G. Callan Jr. and J.A. Harvey. Nuck Phys. **B250** (1985) 427
12. D.B. Kaplan, M. Schmaltz, Phys Lett. **B368** (1996) 44
13. A.G. Cohen, D.B. Kaplan and A.E. Nelson, Ann Rev of Nucl and Part Sci, **43** (1993) 27
14. P. Huet, A.E. Nelson, Phys. Lett. **B355** (1995) 229
15. Vachaspati and T. Vachaspati. Phys. Lett.**B238** (1990) 41
16. J. Dziarmaga Phys Lett **B328** (1994) 392